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INTERLAMINAR STRESSES OF LAMINATED COMPOSITE JOINTS WITH DOUBLE COVER PLATES†

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Abstract—Interlaminar stresses for joints with double cover plates are found by superposing two parts based upon the energy method. As the series encountered can be summed, our solutions are in closed form. The final result is checked exactly by the load the joint has to carry.

1. INTRODUCTION

Let two identical bars of orthotropic material with rectangular cross-section bh_1 be joined lengthwise. They are glued together by two identical metallic cover plates of cross-section bh, forming a laminated composite joint with double cover plates of length $2l$ (Fig. 1). The tensile force P is transmitted to the cover plates by the interlaminar stresses in the adhesive surfaces. These are concentrated near the ends of cover plates and often decide the strength of the joint by causing delamination. E, G, μ are the elastic constants of the metal cover plates. For orthotropic bars, one principal direction is along the length with elastic constant E_1, the others being E_2, G_1 and μ_{12}.

The interlaminar stresses of this joint can be solved by superposing that of the following two parts.

(1) The two bars in Fig. 1 are continuous, forming a laminated composite bar (Fig. 2a) and its interlaminar stresses are the first part. Transforming its cross-section into one of the same material as the cover plates, for $E_1 < E$ we get an I-shaped cross-section (Fig. 2b) with the area equal to $2bh+b(E_1/E)h_1$. At distant cross-sections from the ends of the cover plate, for instance at the middle cross-section, the stresses in the cover plates and bar are, respectively (Fig. 2a)

$$\sigma_x = \frac{P}{2bh+\dfrac{E_1}{E}bh_1}, \quad \sigma'_x = \frac{P}{2bh+\dfrac{E_1}{E}bh_1}\cdot\frac{E_1}{E}. \tag{a}$$

(2) To remove the uniform tensile stress σ'_x in the middle cross-section of the orthotropic bar (Fig. 2a), apply a pair of uniform compressive stresses σ'_x at the two opposite surfaces of the gap in the joint (Fig. 1). Figure 3 shows the left half of the joint, the tensile stress at the middle cross-section of cover plate being $\sigma'_x(h_1/2h)$ to keep it in equilibrium. Its interlaminar stresses form the second part. The corresponding axial compressive and tensile forces are:

$$-\frac{Ph_1}{2h+\dfrac{E_1}{E}h_1}\cdot\frac{E_1}{E}, \quad \frac{Ph_1}{2\left(2h+\dfrac{E_1}{E}h_1\right)}\cdot\frac{E_1}{E}. \tag{b}$$

2. THE INTERLAMINAR STRESSES OF PART ONE

Figure 4 shows the interlaminar stresses τ_0 and σ_0, which are expressed by a sine and cosine series with a_n and b_n to be determined:

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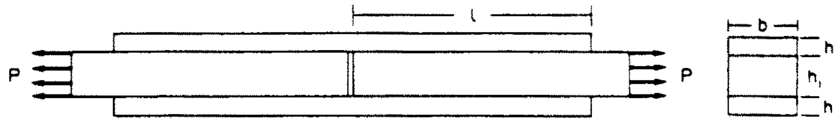

Fig. 1.

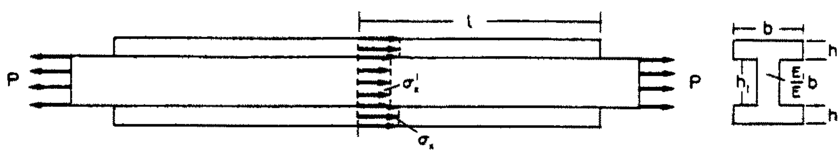

Fig. 2.

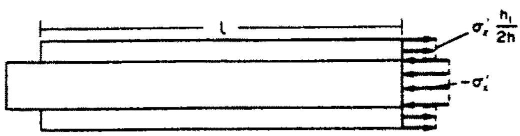

Fig. 3.

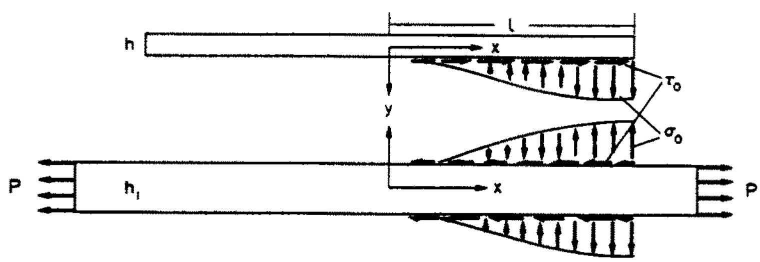

Fig. 4.

$$\tau_0 = \sum_{n=1} a_n \sin \frac{n\pi x}{l}, \quad \sigma_0 = \sum_{n=1} b_n \cos \frac{n\pi x}{l}. \tag{1}$$

As σ_0 will form a couple, $b_0 = 0$.

From the cover plate take an element dx (Fig. 5) upon which are acting the axial force S and shearing force Q. Its equilibrium gives

$$\frac{\mathrm{d}S}{\mathrm{d}x} = -b\tau_0, \quad \frac{\mathrm{d}Q}{\mathrm{d}x} = -b\sigma_0, \quad Q = \frac{\tau_0}{2}bh. \tag{2}$$

From (2) it follows that

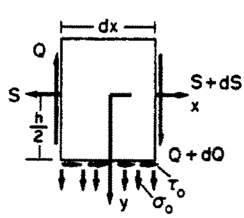

Fig. 5.

$$\sigma_0 = -\frac{h}{2}\cdot\frac{d\tau_0}{dx}. \tag{3}$$

Thus along the adhesive surface σ_0 is proportional to the slope of the τ_0 curve negatively at the very point. From (1) and (3) we have the relation between a_n and b_n:

$$b_n = -\frac{h\pi}{2l}a_n n. \tag{4}$$

It enables us to express the stress components of the cover plates and bar by a_n. Integrating the first equation of (2) we get

$$S = -\frac{bl}{\pi}\sum_{n=1}\frac{a_n}{n}\left(\cos n\pi - \cos\frac{n\pi x}{l}\right).$$

Now the normal stress component σ_x of the cover plate is

$$\sigma_x = \frac{S}{bh} = -\frac{l}{\pi h}\sum_{n=1}\frac{a_n}{n}\left(\cos n\pi - \cos\frac{n\pi x}{l}\right). \tag{5a}$$

Substituting σ_x in the integral from the equilibrium equation

$$\int_y^{h/2}\frac{\partial\sigma_x}{\partial x}\,dy + \int_{\tau_{xy}}^{\tau_0}\frac{\partial\tau_{xy}}{\partial y}\,dy = 0,$$

we get:

$$\tau_{xy} = \left(\frac{1}{2}+\frac{y}{h}\right)\sum_{n=1}a_n\sin\frac{n\pi x}{l}. \tag{5b}$$

And from another equilibrium equation we get

$$\sigma_y = -\left(\frac{1}{4}+\frac{y}{h}+\frac{y^2}{h^2}\right)\frac{h\pi}{2l}\sum_{n=1}a_n n\cos\frac{n\pi x}{l}. \tag{5c}$$

In a similar manner the stress components of the bar are obtained as

$$\begin{aligned}
\sigma'_x &= \frac{P}{bh_1}+\frac{2l}{\pi h_1}\sum_{n=1}\frac{a_n}{n}\left(\cos n\pi - \cos\frac{n\pi x}{l}\right),\\
\tau'_{xy} &= \frac{-2y}{h_1}\sum_{n=1}a_n\sin\frac{n\pi x}{l},\\
\sigma'_y &= -\frac{\pi h}{2l}\left(1+\frac{h_1}{2h}-\frac{2y^2}{hh_1}\right)\sum_{n=1}a_n n\cos\frac{n\pi x}{l}.
\end{aligned} \tag{6}$$

Thus all the stress components are expressed by a_n, which will be determined by the Principle of Least Work. Let U be the total strain energy of this composite bar and the coefficients a_n should be such as to minimize U, namely

$$\frac{\partial U}{\partial a_n} = 2\int_0^l \int_{-h/2}^{h/2} \left\{ \frac{1}{E}\left[\sigma_x \frac{\partial \sigma_x}{\partial a_n} + \sigma_y \frac{\partial \sigma_y}{\partial a_n} - \mu\left(\sigma_x \frac{\partial \sigma_y}{\partial a_n} + \sigma_y \frac{\partial \sigma_x}{\partial a_n} \right) \right] \right.$$

$$\left. + \frac{1}{G}\tau_{xy}\frac{\partial \tau_{xy}}{\partial a_n} \right\} b \, dx \, dy + \int_0^l \int_{-h_1/2}^{h_1/2} \left\{ \frac{\sigma'_x}{E_1}\cdot\frac{\partial \sigma'_x}{\partial a_n} + \frac{\sigma'_y}{E_2}\cdot\frac{\partial \sigma'_y}{\partial a_n} - \frac{\mu_{12}}{E_1}\left[\sigma'_x \frac{\partial \sigma'_y}{\partial a_n} + \sigma'_y \frac{\partial \sigma'_x}{\partial a_n} \right] \right.$$

$$\left. + \frac{\tau'_{xy}}{G_1}\frac{\partial \tau'_{xy}}{\partial a_n} \right\} b \, dx \, dy = 0. \quad (7)$$

Substituting the six stress components in (7) we get, after lengthy calculation,

$$a_n = -C \frac{n \cos n\pi}{n^4 + 2\eta n^2 + p^2}, \quad (8)$$

in which

$$p^2 = \frac{\frac{1}{\pi^2}\left(\frac{1}{E} + \frac{1}{E_1}\frac{2h}{h_1} \right)}{\frac{\pi^2}{4}\frac{h^4}{l^4}\left[\frac{1}{5E} + \frac{1}{E_2}\frac{h_1}{h}\left(\frac{1}{2} + \frac{1}{3}\frac{h_1}{h} + \frac{1}{15}\cdot\frac{h_1^2}{h^2} \right) \right]},$$

$$2\eta = \frac{\frac{1}{3}\frac{\mu}{E} - \frac{\mu_{12}}{E_1}\left(1 + \frac{h_1}{3h} \right) + \frac{1}{3}\left(\frac{1}{G} + \frac{1}{2G_1}\frac{h_1}{h} \right)}{\frac{\pi^2}{4}\frac{h^2}{l^2}\left[\frac{1}{5E} + \frac{1}{E_2}\frac{h_1}{h}\left(\frac{1}{2} + \frac{1}{3}\frac{h_1}{h} + \frac{1}{15}\frac{h_1^2}{h^2} \right) \right]},$$

$$C = \frac{\frac{2}{\pi}\left\{ \frac{1}{E_1}\frac{h}{l}\sigma^* + \frac{1}{\pi}\left(\frac{1}{E} + \frac{2h}{E_1 h_1} \right) \sum_{n=1} \frac{a_n}{n}\cos n\pi \right\}}{\frac{\pi^2}{4}\frac{h^4}{l^4}\left[\frac{1}{5E} + \frac{1}{E_2}\frac{h_1}{h}\left(\frac{1}{2} + \frac{1}{3}\frac{h_1}{h} + \frac{1}{15}\frac{h_1^2}{h^2} \right) \right]}, \quad (9)$$

in which $\sigma^* = P/bh_1$. As can be seen from (9), when the cover plates and bar are of the same isotropic material, p, 2η and C will be free from elastic constants. As a result, τ_0 and σ_0 do not depend upon the elastic constants. When the cover plates are of the same orthotropic material as the bar, as often happens in wooden structures, then in the denominators of (9) E has to be changed into E_2 and in the numerators E, G, μ have to be changed into E_1, G_1, μ_{12}.

From (8) and (1) the interlaminar shearing stresses τ_0 become:

$$\tau_0 = -C \sum_{n=1} \frac{n \cos n\pi \sin \frac{n\pi x}{l}}{n^4 + 2\eta n^2 + p^2}. \quad (10)$$

As series (10) can be summed, τ_0 and σ_0 are both in closed form. For $p > \eta$ and for $\beta\pi$ to be a comparatively large value, as usually occurs τ_0 becomes:

$$\tau_0 = \frac{C\pi}{4\beta\gamma \sinh \beta\pi}\left\{ \sin \gamma\pi \sinh \frac{\beta\pi}{l}x \cos \frac{\gamma\pi}{l}x - \cos \gamma\pi \cosh \frac{\beta\pi}{l}x \sin \frac{\gamma\pi}{l}x \right\}, \quad (11)$$

in which $\beta = \sqrt{\tfrac{1}{2}(p+\eta)}$, $\gamma = \sqrt{\tfrac{1}{2}(p-\eta)}$. From (3), σ_0 is given by

$$\sigma_0 = -\frac{h}{2}\frac{\mathrm{d}\tau_0}{\mathrm{d}x} = \frac{C\pi^2}{4\beta\gamma\sinh\beta\pi}\cdot\frac{h}{2l}\left\{(\gamma\sin\gamma\pi+\beta\cos\gamma\pi)\sinh\frac{\beta\pi}{l}x\sin\frac{\gamma\pi}{l}x\right.$$
$$\left.-(\beta\sin\gamma\pi-\gamma\cos\gamma\pi)\cosh\frac{\beta\pi}{l}x\cos\frac{\gamma\pi}{l}x\right\}. \quad (12)$$

Near the ends of the cover plate when $\sinh(\beta\pi/l)x = \cosh(\beta\pi/l)x$, a further simplification can be made as follows:

$$\tau_0 = \frac{C\pi}{4\beta\gamma}\cdot\frac{\sinh\frac{\beta\pi}{l}x}{\sinh\beta\pi}\sin\gamma\pi\left(1-\frac{x}{l}\right). \quad (13)$$

$$\sigma_0 = \frac{C\pi^2}{4\beta\gamma}\cdot\frac{h}{2l}\frac{\sinh\frac{\beta\pi}{l}x}{\sinh\beta\pi}\left\{\gamma\cos\gamma\pi\left(1-\frac{x}{l}\right)-\beta\sin\gamma\pi\left(1-\frac{x}{l}\right)\right\}. \quad (14)$$

The shearing force transmitted by the cover plate is

$$b\int_0^l \tau_0\,\mathrm{d}x = \frac{Cbh_1}{4p\beta\frac{h_1}{l}}. \quad (15)$$

The value C in the above equations has to be determined from the third equation of (9) which includes the series $\sum_{n=1}(a_n/n)\cos n\pi$. Substituting (8) in it, we have for $p > \eta$ with $\beta\pi$ pretty large:

$$\sum_{n=1}\frac{a_n}{n}\cos n\pi = -C\sum_{n=1}\frac{1}{n^4+2\eta n^2+p^2} = -C\left(\frac{\pi}{4p\beta}-\frac{1}{2p^2}\right). \quad (16)$$

3. THE INTERLAMINAR STRESSES OF PART TWO

Differing from Fig. 4, Fig. 6 corresponds to the left half of the joint with the origin taken at the left end of the cover plate. We still use (1) to denote the interlaminar stresses τ_0 and σ_0.

The equations obtained from the equilibrium of an element $\mathrm{d}x$ of the cover plate are the same as (2). This results in two equations, the same as (3) and (4). The cover plate has its axial force equal to

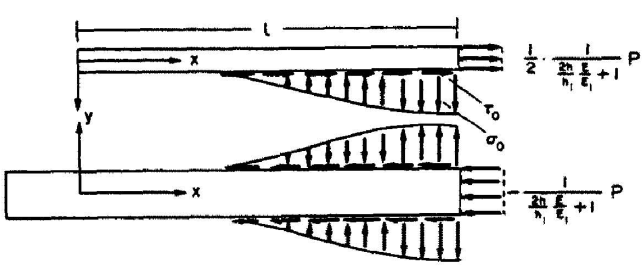

Fig. 6.

$$S = -\frac{bl}{\pi}\sum_{n=1}\frac{a_n}{n}\left(\cos n\pi - \cos\frac{n\pi x}{l}\right) + \frac{1}{2}\frac{P}{\frac{2h}{h_1}\frac{E}{E_1}+1}.$$

The normal stress σ_x is

$$\sigma_x = \frac{S}{bh} = -\frac{l}{\pi h}\sum_{n=1}\frac{a_n}{n}\left(\cos n\pi - \cos\frac{n\pi x}{l}\right) + \frac{\sigma^*}{2}\frac{h_1}{h}\cdot\frac{1}{\frac{2h}{h_1}\frac{E}{E_1}+1}. \tag{c}$$

It differs from (5a) of the first part only in the additional constant term and hence τ_{xy} and σ_y are the same as (5b, c). Similarly, for the bar we have

$$\sigma_x' = \frac{S'}{bh_1} = \frac{2l}{\pi h_1}\sum_{n=1}\frac{a_n}{n}\left(\cos n\pi - \cos\frac{n\pi x}{l}\right) - \frac{\sigma^*}{\frac{2h}{h_1}\frac{E}{E_1}+1}. \tag{d}$$

It also differs from σ_x' of the first part in the constant term, so that τ_{xy}' and σ_y' remain the same as in (6). Substituting the six stress components in (7) we again have a result expressed by equation (8). Besides, of the six stress components only σ_x and σ_x' differ from those of the first part by their constant terms; p and 2η remain the same as in equation (9) of the first part. Only the third one has to be changed, into

$$C = \frac{\frac{2}{\pi}\left\{\frac{\sigma^*}{E_1}\cdot\frac{h}{l}\left(-\frac{h_1E_1}{2hE}\right) + \frac{1}{\pi}\left(\frac{1}{E}+\frac{2h}{E_1h_1}\right)\sum_{n=1}\frac{a_n}{n}\cos n\pi\right\}}{\frac{\pi^2}{4}\frac{h^4}{l^4}\left[\frac{1}{5E}+\frac{1}{E_2}\frac{h_1}{h}\left(\frac{1}{2}+\frac{h_1}{3h}+\frac{1}{15}\frac{h_1^2}{h^2}\right)\right]}. \tag{17}$$

Equation (17) differs from the third part of (9) in that $(\sigma^*/E_1)\cdot(h/l)$ has to be multiplied by a factor

$$-\frac{h_1E_1}{2hE}. \tag{18}$$

Since the two parts of superposition have the same values of p and 2η, their β and γ are also same. Thus from eqns (11) and (12), it can be seen that the two different values of C constitute the difference of two parts of the interlaminar stresses. Apparently, the sum of the series $\sum_{n=1}(a_n/n)\cos n\pi$ within the parentheses of (16) remains the same for the two parts. As a result, from (17) and the third eqn of (9), it is seen that the value of C of the second part is equal to that of first part multiplied by the factor (18). Therefore, the interlaminar stresses of the first part multiplied by this same factor give the interlaminar stresses of second part. Negative τ_0 means the direction is opposite to that shown in Fig. 6 and negative σ_0 means compressive stress.

For cover plates and bar made of the same orthotropic material, by setting E equal to E_1 in (18) we get the factor $-(h_1/2h)$. Multiplying the interlaminar stresses of the first part by this factor, we get the interlaminar stresses of the second part.

However, the solution of first part of superposition is for the right half of the joint, while the second part is for the left half. Owing to the symmetry of σ_0 and anti-symmetry of τ_0, we can easily convert the first part to the left half and superpose with the second part to get the interlaminar stresses of the joint.

4. NUMERICAL ILLUSTRATIVE EXAMPLES

4.1. *Example* 1

An adhesive joint with double cover plates is made of wooden bars and hard aluminium cover plates with $h_1 = 6h$, $l/h = 32$ and $l/(2h+h_1) = l/8h = 4$. For pine bars: $E_1 = 10^5$ kg cm^{-2}, $E_2 = 0.042\times 10^5$ kg cm^{-2}, $G_1 = 0.075\times 10^5$ kg cm^{-2}, $\mu_{12} = 0.238$. For hard aluminium: $E = 7\times 10^5$ kg cm^{-2}, $G = 2.69\times 10^5$ kg cm^{-2}, $\mu = 0.3$.

First part of superposition

From (9) and (11), $p = 5.4121$, $\eta = 3.7816$; $\beta = 2.1440$, $\gamma = 0.90291$. The series sum (16) gives $\sum_{n=1} (a_n/n)\cos n\pi = -0.050616C$. The third equation of (9) gives $C = 3.0459\sigma^*$. By (15) the shearing force transmitted by the cover plate is

$$\int_0^l b\tau_0 \, dx = \frac{3.0459P}{4\times 5.4121\times 2.144\times \frac{6}{32}} = 0.34999P.$$

To check our calculation let us find the tensile force in the middle cross-section of the cover plate. From (a) it is equal to

$$\sigma_x bh = \frac{P}{2+\dfrac{10^5}{7\times 10^5}\times 6} = 0.35P.$$

This indicates the correctness of our calculation. Now by (11)–(14), τ_0 and σ_0 are found and tabulated as follows.

x	l	0.98	0.96	0.94	0.92	0.9	0.85	0.8	0.75
τ_0	0	$0.0710\sigma^*$	0.1069	0.1398	0.1623	0.1763	0.1875	0.1726	0.1491
σ_0	$0.0547\sigma^*$	0.0417	0.0303	0.0213	0.0141	0.0083	−0.0014	−0.0062	−0.0080
x	0.7	0.65	0.6	0.55	0.5	0.45	0.4	0.3	0.2
τ_0	0.1232	0.0979	0.0757	0.0571	0.0421	0.0305	0.0216	0.0102	0.0044
σ_0	−0.0082	−0.0075	−0.0064	−0.0052	−0.0041	−0.0032	−0.0024	−0.0013	

Second part of superposition

The factor (18) is

$$-\frac{6\times 10^5}{2\times 7\times 10^5} = -0.42857.$$

Multiplying τ_0 and σ_0 in the above table by this factor, we get the interlaminar stresses of the second part. The shearing force transmitted by the cover plate (Fig. 6) is equal to the corresponding shearing force of the first part multiplied by this factor, namely

$$-0.42857\times 0.34999P = -0.14999P.$$

The negative sign means its direction is opposite to that in Fig. 6. This result is confirmed by the tensile force at the end of cover plate:

$$\frac{1}{2}\cdot\frac{1}{\dfrac{2}{6}\cdot\dfrac{10^5}{7\times 10^5}+1} = 0.15P.$$

Adding the shearing forces transmitted by the cover plate of two parts, we have the force transmitted by the joint:

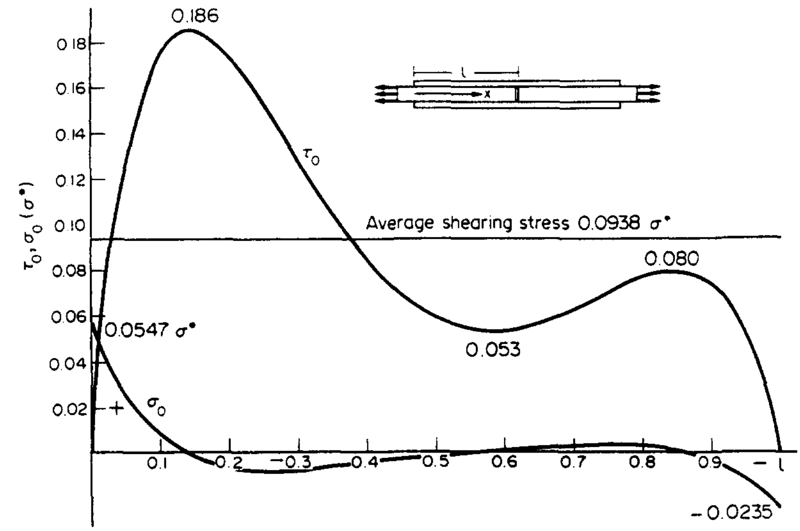

Fig. 7.

$$0.34999P+0.14999P = 0.49998P.$$

The exact value is $0.5P$. Converting τ_0 and σ_0 of the first part to the left half (Fig. 4) and superposing with that of the second part, we get the interlaminar stresses of the joint as shown in Fig. 7.

As can be seen from Fig, 7, σ_0 is small compared with τ_0. The reason is that for the thin hard aluminium cover plate the bending moments formed by τ_0 are rather small and require a comparatively small σ_0 to reduce them to zero. However, the maximum shearing stress is twice as large as the average value.

4.2. *Example* 2

As a second example, let there be a pine adhesive joint with double cover plates, $h/l = 1/16$, $h/h_1 = 3/5$, $l/(2h+h_1) = 4.36$. The elastic constants are as given in first example.

First part of superposition

Converting equations (9) to the case for cover plates and bars made of the same orthotropic material, we get $p = 10.471$, $\eta = 7.5483$. β, γ in (11) are respectively 3.0016 and 1.2089. The series sum $\sum_{n=1} (a_n/n) \cos n\pi = -0.020428C$ and $C = 3.5716\sigma^*$.

To check our solution, use (15) to find the shearing force transmitted by cover plate:

$$\int_0^l b\tau_0 \, dx = \frac{3.5716P}{4\times 10.471\times 3.0016\times \frac{5}{3}\times \frac{1}{16}} = 0.27273P.$$

The tensile force in the middle cross-section of the cover plate is, from (a),

$$\frac{P}{2+\frac{5}{3}} = 0.27273P.$$

The identity of the two results confirms our solution. Now using (11)–(14) we get τ_0 and σ_0 which are tabulated as follows.

x	l	0.98	0.96	0.94	0.92	0.9	0.8	0.75
τ_0	0	0.0486	0.0802	0.0992	0.1088	0.1116	0.0807	0.0595
σ_0	$0.0917\sigma^*$	0.0614	0.0386	0.0215	0.0091	0.0003	−0.0137	−0.0125
x	0.7	0.65	0.6	0.5	0.4			
τ_0	0.0415	0.0277	0.0178	0.0066	0.0020			
σ_0	−0.0099	−0.0073	−0.0051	−0.0022	−0.0008			

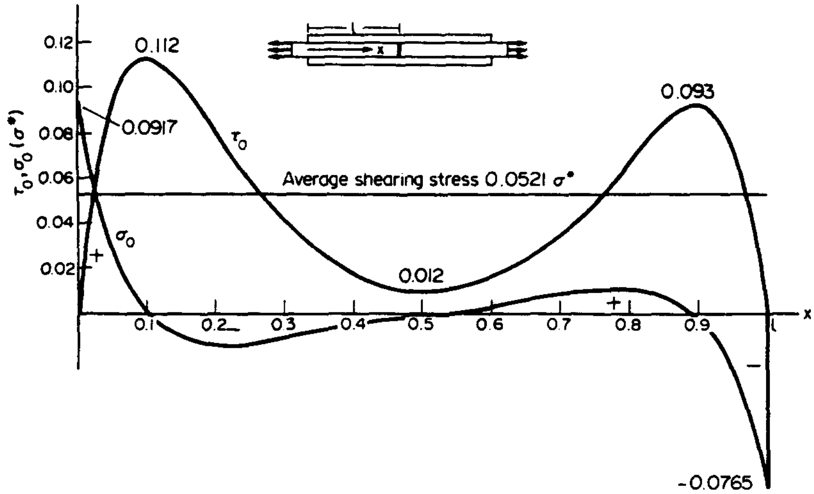

Fig. 8.

Second part of superposition

As factor (18) gives $h_1/2h = -5/6$, multiplying it by the interlaminar stresses of the above table, we get the results of the second part. The shearing force taken by the cover plate along the adhesive surface is

$$-5/6 \times 0.27273P = -0.22727P.$$

This should be equal to the tensile force at the end of the cover plate (Fig. 6):

$$\frac{1}{2} \cdot \frac{1}{2 \times \frac{3}{5} + 1} = 0.22727P.$$

The result obtained confirms our solution. Adding the two shearing forces along the adhesive surfaces of the two parts, we get:

$$0.27273P + 0.22727P = 0.5P.$$

This is exactly equal to the load which a cover plate is required to carry. Superposing the interlaminar stresses of two parts, we have that of the joint as shown in Fig. 8.

The numerical examples show that the interlaminar normal and shearing stresses are concentrated near the ends of the cover plates. This accounts for the tearing of the cover plates from the bar starting at the ends of the cover plates, as observed from experiments with these kinds of joints.

5. DISCUSSION

The interlaminar stresses of laminated composite joints with double cover plates present a problem of practical and theoretical interest. As previous works show, it is still not satisfactorily solved. Yuceoglu and Updike (1980, 1981) regarded the adherends as beams in action, which can take into account only the bending stress σ_x and shearing stress τ_{xy}. The stress component σ_y is, in fact, as important as the normal interlaminar stress σ_0 we are seeking. Disregarding it in adherends leads erroneously to the interlaminar shearing stress τ_0 becoming maximum at the ends of the adhesive surface instead of zero as it should be. Another type of solution was by the finite element method and valid only for special cases of geometry and material.

Our solution is based upon the interlaminar stresses τ_0 and σ_0 of the corresponding laminated composite bar. They are expressed by sine and cosine series as in (1). As the composite bar consists of three slender bars, their stress components σ_x can be obtained by the Mechanics of Materials. The other stress components follow from equations of equilibrium. With the relation between the coefficients a_n and b_n, all the stress components can be expressed by a_n, which is determined by the Principle of Least Work. As all the series encountered can be summed, we get for this composite bar interlaminar stresses in closed form. Seeing that the two series in (1) are complete sets of functions and in the process of investigation we have not neglected anything, exact solutions have been obtained. This confirms the simple solution long taught in Mechanics of Materials and enables us to clarify how far from the ends this simple solution holds. It can also be shown that the interlaminar stresses given here require the composite bar to be sufficiently long, just as the bending theory requires the beam to be rather slender. Furthermore, when the bar is long enough, for any specific composite bar its interlaminar stresses are independent of its length. With all these considerations, we can simply go a step further by superposition to solve the joint problem.

Extensions can be made to solve the following problems:

(1) interlaminar stresses of laminated composite lap joints and joints with a single cover plate;
(2) thermal interlaminar stresses of laminated composite bars and joints;
(3) the effects of an adhesive layer with a certain thickness upon interlaminar stresses of laminated composite bars and joints.

REFERENCES

Erdogan, F. (1971). Analysis of elastic cover plates. *Devel. Mech.* **6**, 817–829.

Hoff, N. J. (1944). Stresses in a reinforced monocoque cylinder under concentrated symmetric transverse loads. *J. Appl. Mech.* **11**, A-235.

Segerlind, L. J. (1968). On the shear stress in bonded joints. *J. Appl. Mech.* **35**, 117–118.

Timoshenko, S. (1924). Approximate solution of two dimensional problems in elasticity. *Philosophical Mag.* **47**, 1095–1104.

Yuceoglu, U. and Updike, D. P. (1980). Stress analysis of bonded plates and joints. *J. Engng Mech. Div. ASCE* **106**, 39–56.

Yuceoglu, U. and Updike, D. P. (1981). Bending and shear deformation effects in lap joints. *J. Engng Mech. Div. ASCE* **107**, 55–76.

Zhang, F. (1985). Interlaminar stresses of laminated composite bars under bending, I, II. *Appl. Maths. Mech.* **7**, **8**, 583–594, 699–710.

Zhang, F. (Chang, F.-v.) (1988). Interlaminar stresss of a laminated composite lap joint. *Scientia Sinica* **31**, 69–78.